Statistična termodinamika 2013/14

2. kolokvij

22. 1. 2014

1. V razredčeno raztopino mehkih kroglic s polmerom 50 nm sta potopljeni vzporedni ravni plošči s površino 1 dm^2, razmaknjeni za 70 nm. Energija interakcije med posamezno kroglico in ploščama je enaka 0, če se kroglica ne prekriva z nobeno izmed njiju, sicer pa znaša 5×10^{-3} eV. Številska gostota kroglic je $10^{16}/\mathrm{m}^3$; interakcije med kroglicami so zanemarljive. Kolikšna sila deluje med ploščama pri 20°C? Koliko dela opravimo, ko razdaljo med ploščama podvojimo?

2. Disperzijska relacija mrežnih nihanj v neki dvorazsežni trdnini je $\omega = \alpha\sqrt{k}$; $\alpha = 2 \times 10^7\ \sqrt{\mathrm{m}}/\mathrm{s}$. Polarizaciji sta dve. Izračunajte prispevek mrežnih nihanj k entropiji vzorca velikosti 1 dm^2 pri 2 K! Kolikšne so tedaj fluktuacije števila fononov (σ_N)?

$$\int_0^\infty \exp(-ax^2)\,\mathrm{d}x = \frac{\sqrt{\pi}}{2\sqrt{a}}$$

$$\int_0^\infty x\exp(-ax^2)\,\mathrm{d}x = \frac{1}{2a}$$

$$\int_0^\infty \frac{x\,\mathrm{d}x}{\exp(x)-1} = \frac{\pi^2}{6}$$

$$\int_0^\infty \frac{x^2\,\mathrm{d}x}{\exp(x)-1} \approx 2.404$$

$$\int_0^\infty \frac{x^3\,\mathrm{d}x}{\exp(x)-1} = \frac{\pi^4}{15}$$

$$\int_0^\infty \frac{x^4\,\mathrm{d}x}{\exp(x)-1} \approx 24.886$$

$$\int_0^\infty \frac{x^5\,\mathrm{d}x}{\exp(x)-1} \approx \frac{8\pi^6}{63}$$

$$\int_0^\infty \frac{x^6\,\mathrm{d}x}{\exp(x)-1} \approx 726.011$$

$$\int_0^\infty \frac{x\,\mathrm{d}x}{\exp(x)+1} = \frac{\pi^2}{12}$$

$$\int_0^\infty \frac{x^2\,\mathrm{d}x}{\exp(x)+1} \approx 1.803$$

$$\int_0^\infty \frac{x^3\,\mathrm{d}x}{\exp(x)+1} = \frac{7\pi^4}{120}$$

$$\int_0^\infty \frac{x^4\,\mathrm{d}x}{\exp(x)+1} \approx 23.331$$

$$\int_0^\infty \frac{x^5\,\mathrm{d}x}{\exp(x)+1} = \frac{31\pi^6}{252}$$

$$\int_0^\infty \frac{x^6\,\mathrm{d}x}{\exp(x)+1} \approx 726.011$$

$$\int_0^\infty x^2\exp(-ax^2)\,\mathrm{d}x = \frac{\sqrt{\pi}}{4\sqrt{a^3}}$$

$$\int_0^\infty x^3\exp(-ax^2)\,\mathrm{d}x = \frac{1}{2a^2}$$

$$\int_0^\infty \frac{x^2\exp(x)}{(\exp(x)-1)^2}\mathrm{d}x = \frac{\pi^2}{3}$$

$$\int_0^\infty \frac{x^3\exp(x)}{(\exp(x)-1)^2}\mathrm{d}x \approx 7.212$$

$$\int_0^\infty \frac{x^4\exp(x)}{(\exp(x)-1)^2}\mathrm{d}x = \frac{4\pi^4}{15}$$

$$\int_0^\infty \frac{x^5\exp(x)}{(\exp(x)-1)^2}\mathrm{d}x \approx 124.431$$

$$\int_0^\infty \frac{x^6\exp(x)}{(\exp(x)-1)^2}\mathrm{d}x = \frac{16\pi^6}{21}$$

$$\int_0^\infty \frac{x^7\exp(x)}{(\exp(x)-1)^2}\mathrm{d}x \approx 5082.08$$

$$\int_0^\infty \frac{x\exp(x)}{(\exp(x)+1)^2}\mathrm{d}x = \ln(2)$$

$$\int_0^\infty \frac{x^2\exp(x)}{(\exp(x)+1)^2}\mathrm{d}x = \frac{\pi^2}{6}$$

$$\int_0^\infty \frac{x^3\exp(x)}{(\exp(x)+1)^2}\mathrm{d}x \approx 5.409$$

$$\int_0^\infty \frac{x^4\exp(x)}{(\exp(x)+1)^2}\mathrm{d}x = \frac{7\pi^4}{30}$$

$$\int_0^\infty \frac{x^5\exp(x)}{(\exp(x)+1)^2}\mathrm{d}x \approx 116.654$$

$$\int_0^\infty \frac{x^6\exp(x)}{(\exp(x)+1)^2}\mathrm{d}x = \frac{31\pi^6}{42}$$