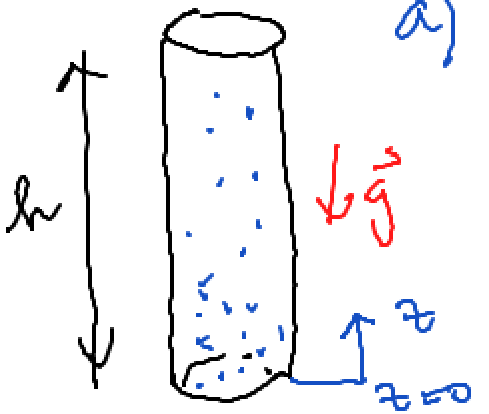
(1) Valj s plinom (idealnim) v težnostnem polju ($h = 5\,\mathrm{km}$, $T = 283\,\mathrm{K}$, $M = 32\,\frac{\mathrm{kg}}{\mathrm{kmol}}$; $g = 10\,\frac{\mathrm{m}}{\mathrm{s}^2} = \text{konst.}$)



a) $\rho(z) \propto e^{-\beta E(z)} = e^{-\beta m g z}$;

↑ verjetnostna gostota

$\langle z \rangle = \frac{\langle E \rangle}{mg}$; $\sigma_z = \sqrt{\langle z^2\rangle - \langle z \rangle^2} = \frac{\sqrt{\langle E^2\rangle - \langle E\rangle^2}}{mg} = \frac{\sigma_E}{mg}$ *

samo potencialna!

$$e^{-\beta F} = C \int_0^h e^{-\beta m g z}\, dz = C\,\frac{(1 - e^{-\beta m g h})}{\beta m g}$$

$$+\beta F = \text{konst.} + \ln\beta - \ln(1 - e^{-\beta m g h})$$

$$\langle E \rangle = \frac{d\beta F}{d\beta} = \frac{1}{\beta} - \frac{(-mgh)\, e^{-\beta m g h}}{1 - e^{-\beta m g h}}$$

$$\langle z \rangle = \frac{\langle E \rangle}{mg} = h\left(\frac{1}{\beta m g h} - \frac{1}{e^{\beta m g h} - 1}\right) = 2{,}23\,\mathrm{km}$$ 1/4+

$$\beta m g h = \frac{32\,\mathrm{kg} \cdot 10\,\frac{\mathrm{m}}{\mathrm{s}^2} \cdot 5000\,\mathrm{m}}{6\cdot 10^{26} \cdot 1{,}38\cdot 10^{-23}\,\mathrm{J/K} \cdot 283\,\mathrm{K}} = 0.6595$$

b) $$\sigma_E^2 = -\frac{d\langle E\rangle}{d\beta} = \frac{1}{\beta^2}\left(1 - \frac{(\beta m g h)^2 e^{\beta m g h}}{(e^{\beta m g h} - 1)^2}\right)$$

$$\sigma_z = \frac{\sigma_E}{mg} = \frac{1}{\beta m g}\sqrt{\cdots} = 1{,}43\,\mathrm{km}$$ 1/4+

c) povprečna prosta pot $l_p = \frac{1}{\sqrt{2}\, n \pi \sigma^2}$

$$\frac{l_p(h)}{l_p(0)} = \frac{n(0)}{n(h)} = \frac{\cancel{N}\cdot\rho(0)}{\cancel{N}\cdot\rho(h)} = e^{\beta m g h} = 1{,}93$$ 1/4+

↖ številska gostota

ALTERNATIVNO:

$$\langle z \rangle = \frac{\int_0^h z\, e^{-\beta m g h}\, dz}{\int_0^h e^{-\beta m g h}\, dz} = \frac{I_1}{I_0} \;(= 2{,}23\,\mathrm{km}) \;;\quad \langle z^2 \rangle = \frac{\int_0^h z^2 e^{-\beta m g h}\, dz}{\int_0^h e^{-\beta m g h}\, dz} = \frac{I_2}{I_0} \;(= 7{,}00\,\mathrm{km}^2)$$

→ * $\sigma_z = 1{,}43\,\mathrm{km}$

integrala I_0, I_1, I_2 izračunamo (per partes) ali najdemo v tabelah (Bronštejn)

② Magnoni v 3D feromagnetu: $\omega(k) = a k^2$; $a = 7{,}6 \cdot 10^{-6} \frac{m^2}{s}$; $T = 1K$

(μ=0, bozoni)

a) $\langle N \rangle = \sum_j \frac{1}{e^{\beta\hbar\omega_j} - 1}$; $\sum_j 1 = \int \frac{d^3r\, d^3p}{h^3} = \frac{V}{h^3}\int 4\pi p^2 dp \underset{p=\hbar\sqrt{\frac{\omega}{a}}}{=} \frac{V}{4\pi^2}\cdot\frac{\omega^{1/2}}{a^{3/2}} d\omega$

$$\frac{\langle N \rangle}{V} = \frac{1}{4\pi^2 a^{3/2}} \int_{\omega_{min} \to 0 \text{ (velik vzorec)}}^{\omega_{max}} \frac{\omega^{1/2} d\omega}{e^{\beta\hbar\omega} - 1} = \frac{1}{4\pi^2}\frac{1}{(\beta\hbar a)^{3/2}} \underbrace{\int_0^{\beta\hbar\omega_{max} \to \infty \text{ (majhen } T)} \frac{u^{1/2} du}{e^u - 1}}_{I} = \frac{1}{4\pi^2}\left(\frac{1{,}38\cdot 10^{-23} \cancel{J} \cdot 1 \cancel{K} \cdot 2\pi}{\cancel{K} \cdot 6{,}62\cdot 10^{-34} \cancel{Js} \cdot 7{,}6 \cdot 10^{-6} \frac{m^2}{\cancel{s}}}\right)^{3/2} \cdot 2{,}315 =$$

$$= 1{,}32 \cdot \frac{10^{23}}{m^3} \quad 1/2$$

$$I = \int_0^\infty \frac{u^{1/2} e^{-u} du}{1 - e^{-u}} = \int_0^\infty u^{1/2} \sum_{n=1}^\infty e^{-\overset{z}{nu}} du = \underbrace{\sum_{n=1}^\infty \frac{1}{n^{3/2}}}_{\zeta(3/2)} \underbrace{\int_0^\infty z^{1/2} e^{-z} dz}_{\Gamma(3/2)\ *} = 2{,}315$$

b) entropija: $S = -\left(\frac{\partial F}{\partial T}\right)_{V,N} = -\left(\frac{\partial q}{\partial T}\right)_{V,N}$

$\beta q = \beta(F - \underset{0}{\mu N}) = \beta F$

$$\beta q = \sum_j \ln(1 - e^{-\beta\hbar\omega_j}) = \frac{V}{4\pi^2} \int_{\omega_{min} \to 0}^{\omega_{max}} \frac{\omega^{1/2}}{a^{3/2}} \ln(1 - e^{-\beta\hbar\omega}) d\omega = \frac{V}{4\pi^2 (\beta\hbar a)^{3/2}} \underbrace{\int_0^{\beta\hbar\omega_{max} \to \infty} \ln(1 - e^{-u})\sqrt{u}\, du}_{I'}$$

$$= -\frac{V \zeta(\frac{5}{2})\Gamma(\frac{3}{2})}{4\pi^2 (\hbar a)^{3/2}} \cdot k_B^{3/2} T^{3/2}$$

$$I' = \int_0^\infty \sqrt{u} \cdot \sum_{n=1}^\infty (-1) \cdot \frac{e^{-nu}}{n} du = -\sum_{n=1}^\infty \frac{1}{n^{5/2}} \cdot \Gamma\left(\frac{3}{2}\right) = -\zeta\left(\frac{5}{2}\right)\Gamma\left(\frac{3}{2}\right) = -1{,}188$$

$$\frac{S}{V} = -\left(\frac{\partial q}{\partial T}\right)_{V,N} \frac{1}{V} = \frac{5 k_B}{8\pi^2} \frac{1}{(\beta\hbar a)^{3/2}} \zeta\left(\frac{5}{2}\right)\Gamma\left(\frac{3}{2}\right) = 2{,}35 \frac{J}{Km^3} \quad 1/2$$

Ali: $S = \frac{\langle E \rangle - F}{T}$; $\langle E \rangle = \sum_j \frac{\hbar\omega_j}{e^{\beta\hbar\omega_j} - 1} + \ldots$